Chapter 3: Derivatives

3.1: Tangents and the derivative at a point

Def^n: The slope of the curve $y=f(x)$ at the point $P(x_0, f(x_0))$ is the number

$$m := \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} \qquad \text{(if the limit exists).}$$

The tangent line to the curve at P is the line through P with this slope. m is also called the instantaneous rate of change. This is also called the derivative at a point x_0.

Ex(1): $f(x)=\frac{1}{x}$. Where does the slope equal $-\frac{1}{4}$? $a=\pm 2$
What is the tangent line at this point?

Ex(2): $f(t)=16t^2$. What is the derivative at $t=1$? $f'(1)=32$.

3.2: The derivative as a function:

Def^n: The derivative of a fcn ~~with~~ $f(x)$, with respect to the variable x, is denoted f' whose value at x is

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$

Taking a derivative is called differentiation.

Ex(1): $f(x) = \frac{x}{x-1}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h} = \lim_{h \to 0} \frac{(x+h)(x-1) - x(x+h-1)}{h\,(x+h-1)(x-1)}$$

$$= \frac{-1}{(x-1)^2}.$$

Ex(2): $f(x) = \sqrt{x}$ $(x > 0)$.

(a) Rewrite $f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{z - x} = \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{(\sqrt{z} - \sqrt{x})(\sqrt{z} + \sqrt{x})}$

$$= \frac{1}{2\sqrt{x}}$$

(b) Tangent line at $x = 4$ has slope $f'(4) = \frac{1}{4}$ and goes through $(4, 2)$

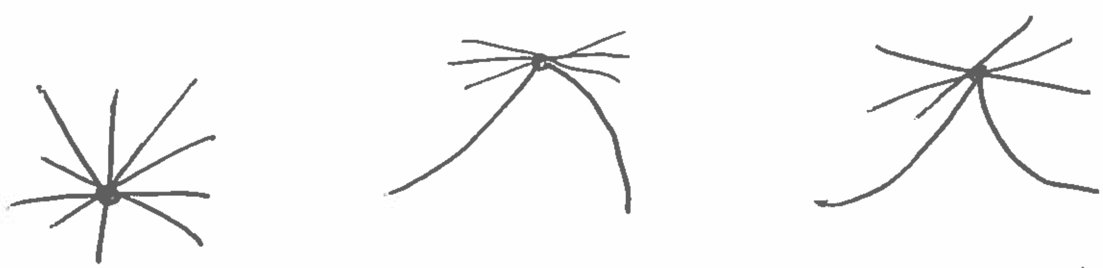
So $y = \frac{1}{4}x + 1$.

Notations: $f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = D_x f(x)$.

Remark: We can define one-sided derivatives in an obvious way, but will not spend time on it.

When does a fcn not have a derivative?

$|x|$



Too many possible tangent lines.

Theorem: If a fcn is differentiable, it is continuous.